

# Wigner delay time from a random passive and active medium

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## Abstract

We consider the scattering of electron by a one-dimensional random potential (both passive and active medium) and numerically obtain the probability distribution of Wigner delay time ( $\tau$ ). We show that in a passive medium our probability distribution agrees with the earlier analytical results based on random phase approximation. We have extended our study to the strong disorder limit, where random phase approximation breaks down. The delay time distribution exhibits the long time tail ( $1/\tau^2$ ) due to resonant states, which is independent of the nature of disorder indicating the universality of the tail of the delay time distribution. In the presence of coherent absorption (active medium) we show that the long time tail is suppressed exponentially due to the fact that the particles whose trajectories traverse long distances in the medium are absorbed and are unlikely to be reflected.

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The concept of delay time which was introduced by Wigner [1] way back in 1955 has received renewed interest in recent years [2–8]. The delay time in the scattering process is generally taken to be related to the duration of collision event or the time spent by the particle (the wave packet) in the region of interaction. Very recently universal parametric correlations of phase shifts and delay times in quantum chaotic scattering regimes have been reported [5]. By assuming the validity of random matrices for describing the statistics of closed chaotic systems the distribution of inverse delay time is given by the Laguerre ensemble from random matrix theory. The delay time statistics is intimately connected with the issue of dynamic admittance of microstructures [9]. The case study being the mesoscopic capacitance [10] fluctuation for the case of a chaotic cavity coupled capacitively to the backgate [6]. This has involved calculation of delay time given by the energy derivative of the phase associated with the scattering matrix.

It may be noted that for the case of a single channel the distribution of the delay time for a disordered semi-infinite sample has been obtained earlier by using the invariant imbedding approach [2]. The stationary distribution  $P_s(\tau)$  for dimensionless delay time  $\tau$  is given by

$$P_s(\tau) = \frac{\lambda e^{\lambda \tan^{-1} \tau}}{(e^{\lambda \pi/2} - 1)(1 + \tau^2)}, \quad (1)$$

where  $\lambda$  is proportional to the disorder induced localization length and the most probable value of  $\tau$  occurs at  $\tau_{max} = \lambda/2$ . At long time tail of the above distribution scales as  $1/\tau^2$ . The average value of  $\tau$  is logarithmically divergent indicating the possibility of the particle traversing the infinite sample before being totally reflected, presumably due to the Azbel resonances [11], which makes Landauer's four probe conductance infinite even for a finite sample. If the disordered region is semi-infinite, the reflection coefficient will be unity, and the complex reflection amplitude will have the form  $R = e^{i\theta(E)}$ . If the wave packet is incident on the disordered sample it will not be immediately reflected back into the lead region, but will be delayed by time proportional to  $\tau = \hbar d\theta/dE$ . This energy dispersive backscattering (or energy dependent random time delay) leads to non-cancellation of the instantaneous currents at the surface involving the incident and reflected waves. This

non-cancellation is expected to lead to a low temperature  $1/f$  type noise that should be universal [2–4]. A very recent study claims the delay time distribution in one-channel case to be universal and is independent of the nature of disorder (especially the long time tail distributions) [7]. We would like to examine this claim through our study. We would like to emphasize that to obtain  $P_s(\tau)$  (Eqn. 1) earlier studies invoke several approximations such as random phase approximation (RPA), which is only valid in the small disorder regime and moreover, correlation between the phase and the delay time is neglected.

In view of the recent claim of universality in the delay time distribution, we have studied numerically the one-dimensional disordered medium and computed the  $P_s(\tau)$ . Our results agree qualitatively with those obtained earlier even in the strong disorder limit where random phase approximation breaks down. We also observe same scaling behavior of the tail of the stationary delay time distribution for different distribution of the disorder potential. We have also studied the delay time distribution in a random active medium, i.e., in the presence of coherent absorption. In this case we observe that  $1/\tau^2$  long time tail distribution is strongly suppressed. This is due to the fact that in the absence of absorption the tail distribution arises from the possibility of particle traversing the infinite sample (due to resonances) before being reflected [2,11]. As these paths traverse a long distance, in the presence of absorption the particles will get absorbed in the medium and they cannot be reflected. Consequently distribution at the tail gets exponentially suppressed. There are several physical situations where one encounters the absorption of elementary particles (electrons, excitons or magnetic excitations, etc.) due to impurities or trapping centers in a medium. One recent example being the light (photon) propagation in a lossy dielectric medium [12–14].

To describe the motion of a quasi particle on a lattice, we use the following Hamiltonian in a tight-binding one-band model:

$$H = \sum \epsilon_n |n\rangle \langle n| + V(|n\rangle \langle n+1| + |n\rangle \langle n-1|), \quad (2)$$

where  $|n\rangle$  is the non degenerate Wannier orbital at site  $n$ ,  $\epsilon_n$  is the site energy at the site  $n$  and  $V$  is the hopping matrix element connecting nearest neighbors separated by

a unit lattice spacing. We consider three different kinds of disorder where the site energies  $\epsilon_n$  are assumed uncorrelated random variables having distributions which are uniform ( $P(\epsilon_n) = 1/W$ ), Gaussian ( $P(\epsilon_n) \propto e^{-\epsilon_n^2/W^2}$ ) and exponential ( $P(\epsilon_n) \propto e^{-\epsilon_n/W}$ ). In case of modeling absorption we make the site energies complex ( $\epsilon_n \equiv \epsilon_n + i\eta$ ) with the imaginary part  $\eta$  chosen to be spatially constant (coherent absorption) [12]. The  $N$  site ( $n = 1$  to  $N$ ) disordered 1D sample is embedded in a perfect infinite lattice having all site energies zero. The well known transfer-matrix method [12,15,16] is used to calculate the reflection amplitude  $r(E) = |r|e^{-i\theta(E)}$  and its phase  $\theta(E)$  at two values of incident energy  $E = E_0 \pm \delta E$ . The delay time is then calculated using the definition  $\tau = \hbar d\theta/dE$ . Throughout our following discussion we consider delay time  $\tau$  in a dimensionless form by multiplying it with  $V$  and we set  $\hbar = m = 1$ .

In view of the fact that the value of incident energy  $E_0$  will not change the physics of the problem, in the following we choose  $E_0 = 0$  and  $dE = 2\delta E = 0.002$ . In calculating the distribution of various quantities we have taken at least 10 000 realizations of the disordered sample. The disorder strength ( $W$ ) and absorption strength ( $\eta$ ) are scaled with respect to  $V$  i.e.,  $W \equiv W/V$  and  $\eta \equiv \eta/V$ . To calculate the stationary distribution of delay time we have considered a sample of length ( $L$ ) 5 times the localization length ( $\xi$ ), where the localization length is calculated by a standard prescription [17]. We have confirmed that by changing the length  $L$  of the sample the distribution remains unchanged.

In Fig. 1 (a) and (b) we show the plot of numerical data (thin line) for the stationary distribution  $P_s(\tau)$  of delay time  $\tau$  for weak disorder ( $W = 0.5$ ) and strong disorder ( $W = 2.0$ ) respectively. The thick line in the figure is the numerical fit obtained by using the expression for  $P_s(\tau)$  given in Eqn. 1. We see that the fit is fairly good even for strong disorder ( $W = 2.0$ ) for which the stationary distribution of the phase of reflected wave,  $P_s(\theta)$ , shows (inset of Fig. 1 (b)) two distinct peaks indicating the failure of random phase approximation (RPA) in this regime [18,19]. The values of  $\lambda$  thus obtained, when plotted against  $1/W^2$ , indicate that  $\lambda$  scales as  $1/W^2$  as shown in the Fig. 2. Since the energy of incoming particles is fixed, the most probable traversal time will be proportional to the typical length traveled by the

particle in the sample, i.e., to the localization length. As the localization length for passive disordered systems [17] scales as  $1/W^2$  and  $\lambda = \tau_{max}/2$ , the proportionality of  $\lambda$  to  $1/W^2$  stands clear. In the inset of Fig. 2 we show the plot of  $\lambda$  versus  $1/W^2$  for Gaussian and exponential disorder. We see that in the weak disorder limit  $\lambda$  scales as  $1/W^2$  for Gaussian and exponential disorder as well.

We would now like to take a closer look at the tail of the stationary distribution  $P_s(\tau)$ . As mentioned earlier, within RPA the long time delay distribution would scale as  $1/\tau^2$ . The numerical least square fit of the expression  $\alpha/\tau^\beta$  to the long time tail data gives  $\beta \approx 2$ . This is shown in Fig. 3. The appearance of such a tail is attributed to the presence of resonant states is due to certain realizations, as in these cases the particle travels a long distance before getting reflected [2].

Motivated by the recent claim [7] of universality of delay time distribution in one-channel case (within RPA), we numerically study the delay distribution  $P_s(\tau)$  for the case of Gaussian and exponential disorder. The expression in Eqn. 1 was derived within RPA using a Gaussian, delta-correlated random potential with zero mean. We have already seen that it fits very well with the data for  $P_s(\tau)$  in case of uniform disorder. Similarly, The agreement of Eqn. 1 with numerical data for Gaussian and exponential disorder is also excellent in the weak disorder limit.

We now look at the tail of delay distribution and its universality for the three different kinds of disorder beyond RPA. Since the origin of tail is due to the appearance of resonant realizations which are independent of strength and the type of disorder, we expect that the tail distribution would be universal beyond RPA too. In Fig. 4 we plot the tail distribution of  $P_s(\tau)$  for uniform, Gaussian and exponential disorder characterized by the strength  $W = 1.0$ . It can be readily noticed that in all the cases rescaled graphs fall on same curve (within our numerical error) clearly indicating the universal nature of tail distribution. For the value  $W = 1.0$ , we are in a regime beyond RPA as can be seen from the non-uniformity of the stationary distribution  $P_s(\theta)$  of the phase of the reflected wave shown in the inset of the Fig. 4. Therefore, our numerical simulation results suggest the existence of universality in

the long time tail distribution beyond RPA.

Finally we consider the stationary delay distribution for a coherently absorbing random medium. We take the absorption strength  $\eta$  to be 0.1 and the disorder strength  $W$  to be 1.0 for the active random medium. From Fig. 5 we see that the delay distribution for active (coherently absorbing) medium (shown with thick line) falls off much rapidly as compared to that of passive medium (shown with thin line). The numerical fit of the tail distribution for the active medium, shown in the inset of Fig. 5, indicates that the fall off is exponential as  $Ae^{-\alpha\tau}/\tau^{2.5}$ . This can be interpreted in terms of suppression of the resonant realizations due to the presence of absorption. In case of these resonant realizations, the time spent by the particle inside the sample is large as it travels large distance before getting reflected. This enhances the probability of the particle getting trapped or absorbed (exponentially) in the traversal length thereby suppressing the tail. Analytical study of the distribution of delay time in the presence of coherent absorption is under active consideration by us.

In conclusion, we have numerically studied the universality of the long time tail of the stationary delay time distribution for a disordered one dimensional sample. Our study reveals that the universality (with respect to the type of disorder) holds for weak as well as strong disorder, i.e., irrespective of the validity of the RPA. Also the typical long tail ( $\sim 1/\tau^2$ ) in the stationary distribution gets exponentially suppressed if we consider a coherently absorbing random medium.

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# FIGURES

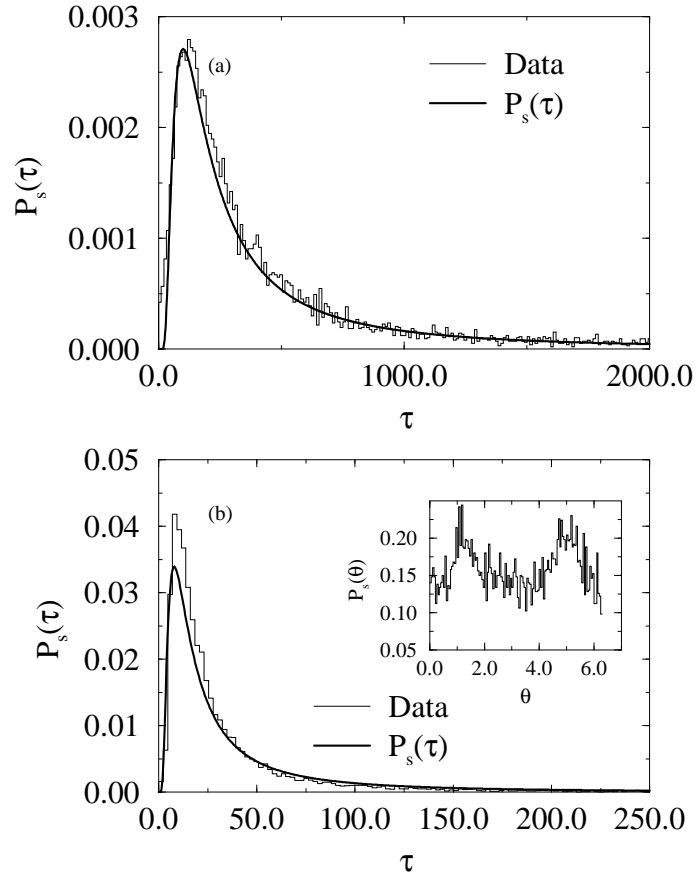


FIG. 1. The stationary distribution of delay time  $P_s(\tau)$  for (a) weak disorder ( $W=0.5$ ) and (b) strong disorder ( $W=2.0$ ).

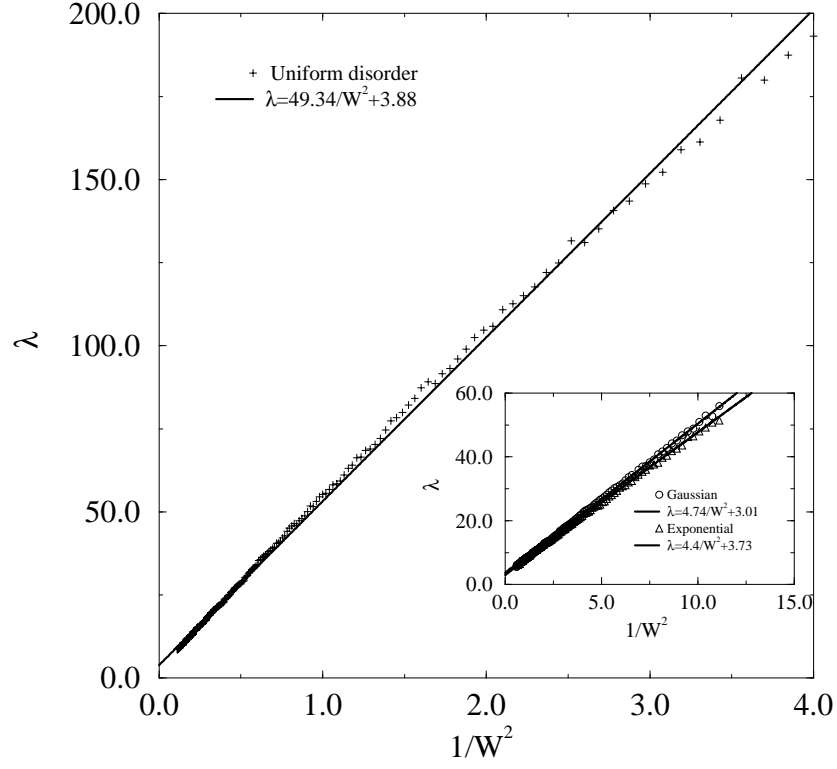


FIG. 2. Plot of  $\lambda$  versus  $1/W^2$  for uniform disorder. The numerical fit is valid only for small values of disorder strength i.e.,  $W \ll 1.0$ . The inset shows the plot of  $\lambda$  versus  $1/W^2$  for Gaussian and exponential disorder.

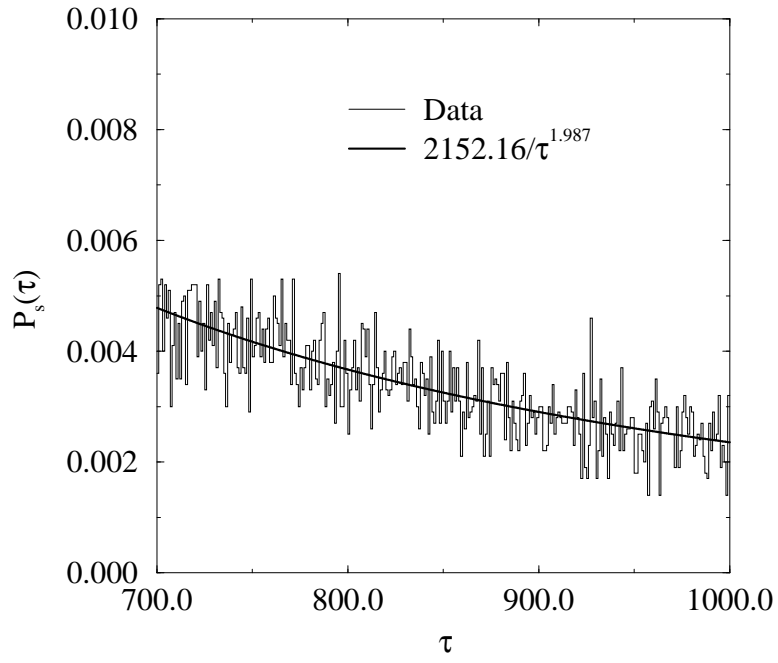


FIG. 3. The plot of tail of  $P_s(\tau)$  for  $W = 0.5$ . Numerical fit (thick line) using expression  $\alpha/\tau^\beta$  yields  $\beta \approx 2.0$ .

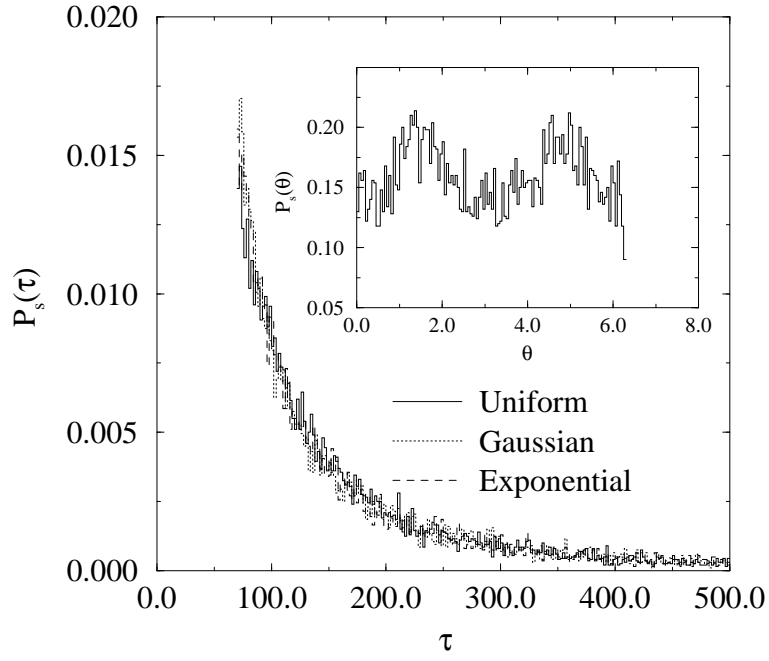


FIG. 4. The plot of tail of  $P_s(\tau)$  for the case of uniform, Gaussian and exponential disorder. The disorder strength in all three cases is  $W = 1.0$ .

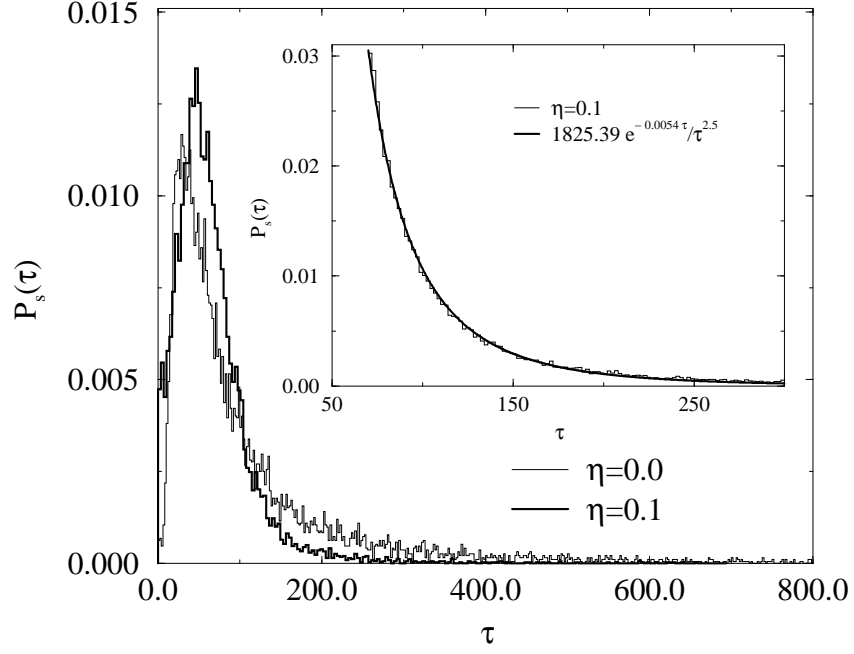


FIG. 5. The plot of  $P_s(\tau)$  for passive ( $\eta = 0$ , shown with thin line) and active ( $\eta = 0.1$ , shown with thick line) disordered medium. The inset shows the tail of  $P_s(\tau)$ , numerical data using thin line and numerical fit using thick line, for the absorbing medium.